RESEARCH ARTICLE

Distributed cross-layer optimization for wireless regional area network-based cognitive radio networks
Wenxuan Guo* and Xinming Huang

Department of Electrical and Computer Engineering, Worcester Polytechnic Institute, Worcester, MA 01609-2280, U.S.A.

ABSTRACT

This paper presents a study of a cross-layer design through joint optimization of spectrum allocation and power control for cognitive radio networks (CRNs). The spectrum of interest is divided into independent channels licensed to a set of primary users (PUs). The secondary users are activated only if the transmissions do not cause excessive interference to PUs. In particular, this paper studies the downlink channel assignment and power control in a CRN with the coexistence of PUs and secondary users. The objective was to maximize the total throughput of a CRN. A mathematical model is presented and subsequently formulated as a binary integer programming problem, which belongs to the class of non-deterministic polynomial-time hard problems. Subsequently, we develop a distributed algorithm to obtain sub-optimal results with lower computational complexity. The distributed algorithm iteratively improves the network throughput, which consists of several modules including maximum power calculation, excluded channel sets recording, base station throughput estimation, base station sorting, and channel usage implementation. Through investigating the impacts of the different parameters, simulation results demonstrates that the distributed algorithm can achieve a better performance than two other schemes. Copyright © 2011 John Wiley & Sons, Ltd.

KEYWORDS

cross-layer design; cognitive radio networks; distributed optimization

*Correspondence
Wenxuan Guo, Department of Electrical and Computer Engineering, Worcester Polytechnic Institute, 100 Institute Road, Worcester, MA 01609-2280, U.S.A.
E-mail: wenxuanguo@gmail.com

1. INTRODUCTION

Over the past decade, the demand for wireless spectrum use has been growing rapidly because of the dramatic development of the mobile telecommunication industry. According to the traditional access approach, the spectrum is divided into fixed portions which are assigned to license holders for exclusive use. As a result, although many licensed portions of a spectrum remain under-utilized, a lot of unlicensed wireless users are prevented to access the wireless media. Therefore, the traditional spectrum allocation can be very inefficient.

In order to fully utilize the scarce spectrum resources, emerging cognitive radio technology becomes a promising approach to exploit the under-utilized spectrum [1]. In a cognitive radio network (CRN), unlicensed wireless users (secondary users [SUs]) are allowed to dynamically access the licensed bands, as long as the licensed wireless users (primary users [PUs]) in those particular bands are not interfered. Wireless devices equipped with cognitive radios, which render more efficient use of available spectrum, are implemented with flexibility, including frequency agility, transmit power control, access coordination, and so on.

In this paper, we consider a CRN based on wireless regional area networks (WRANs) [2] that consists of several secondary base stations (BSs) and SUs. A primary network, consisting of several primary BSs and PUs, coexists within the same area. The spectrum of interest, which is licensed to PUs, is divided into a set of multiple orthogonal channels using frequency division multiple access (FDMA). We assume that the channel usage pattern for PUs is fairly static over time so that CRNs have ample time to implement PU detection and thereby avoid interfering with PUs’ communications. We consider a downlink scenario in the CRN. Each WRAN BS employs exactly one channel to support an SU. An SU can be active or idle indicating whether it is supported or not.

Cognitive radio networks may operate in infrastructure-based systems. As a practical application, IEEE 802.22
Distributed cross-layer optimization

W. Guo and X. Huang

WRAN dynamically allocates TV spectrum to SUs [2] while keeping provisioning service to PUs. The TV bands are selected because they feature very favorable propagation characteristics and are scarcely used because of the popularity of cable and satellite TV services. Therefore, within this overlaid WRAN system, CRN must avoid interfering with PUs as the licensed TV customers by carefully controlling its transmit power and channel selection.

The CRN throughput, defined as the sum of the rates of all channels supporting SUs, is an important performance measure. In our proposed CRN model, the CRN throughput is highly dependent on the channel assignment of SUs as well as on the rate of each channel. For a multi-cell network, because of spatial multiplexing, the network throughput can be greatly improved if multiple users can transmit simultaneously on the same channel within the interference threshold.

In this paper, we study the joint problem of power control and channel assignment to maximize CRN throughput. Channel assignment exerts a great role in CRN throughput performance. After assigning channels to all PUs, channel assignment for SUs should be carefully designed to avoid interfering all other users. In addition, the transmit power control of CRN, constrained by channel assignment of primary network, influences the interference noise powers from transmissions of SUs, thus affects the channel assignment of SUs and thereby the cognitive network throughput. Besides, transmit powers of BSs in CRN greatly impacts the rates of channels supporting SUs. Therefore, channel assignment and power control in CRN are both of great importance to CRN throughput.

Joint study of power control on physical layer and channel assignment on data link layer falls into the class of cross-layer design problems. We consider the power control parameter as a discrete variable, for example, a finite number of equally spaced power levels. As a result, the network throughput problem formulation falls into the integer programming problems, which can be solved by the branch-and-bound algorithm [3]. Although this algorithm guarantees the optimal solution, the complexity could be as high as $O(2^M)$ ($M$ denotes the number of binary variables). Subsequently, we put forward a distributed algorithm, under the condition that each BS is only aware of the channel usage pattern in its vicinity. Note that the order of channel assignment can be critical because later-assigned channels should neither interfere with nor be interfered by previously assigned channels. Therefore, the distributed algorithm is implemented in a greedy fashion in the interest of enhancing the CRN throughput.

The main contribution of this paper are as follows:

- An overlaid CRN is constructed with an existing primary network. We model the opportunistic spectrum access for CRN to formulate the cross-layer optimization problem under the interference constraint imposed by the existing primary network.
- A distributed greedy algorithm is proposed to approximate the optimal network throughput. Cross-layer optimization for CRN is often implemented in a centralized manner to avoid co-channel interference. The distributed algorithm coordinates the channel assignment with local channel usage information. Thus, the computational complexity is greatly reduced.

The rest of this paper is organized as follows. In Section 2, we review the related work about channel assignment and power control in CRNs. In Section 3, we describe the network and interference models and formulate the CRN maximizing throughput problem. Section 4 describes the distributed scheme as a near-optimal low-complexity algorithm. In Section 5, we present some other related algorithms. In Section 6, simulation results are provided to compare the performances between the distributed and the optimal algorithms, followed by some discussion in Section 7 and conclusion in Section 8.

2. RELATED WORK

This paper provides a detailed study of the problem partially addressed in our earlier work [4]. Regarding channel assignment and power control for CRNs, our work is related with [5–10]. In [7], Hoang and Liang considered the joint problem of downlink channel assignment and power control for CRN. Our problem is different from that because our objective is to achieve the maximum network throughput instead of supporting the most number of SUs. In [8], the problem is studied to maximize the throughput for a CRN while not affecting the performance of PUs. In [10], the problem of selecting the maximum subset of SUs to maximize the total secondary revenue of the CRN is investigated, while the quality-of-service requirements for both PUs and admitted SUs should be guaranteed. In [6], a joint problem of optimizing power control and channel assignment is considered for CRNs. However, only one BS is introduced to control and support a set of SUs at fixed locations for all the works mentioned. In our network model, multiple BSs are jointly considered to support the SUs, which results in inter-cell interference and thus leads to a more complex problem to deal with. In [9], Shi and Hou studied the problem of channel allocation, power control, and routing assignment for multi-hop CRNs and proposed a distributed algorithm. The difference with our work mainly lies in the fact that traffic flows from a set of BSs to an unknown subset of user nodes in our network model, whereas the destination nodes are pre-specified in [9]. In [5], a distributed cross-layer optimization scheme, which incorporates scheduling, power control, and channel assignment, is proposed. However, interference constraint is not considered in the problem formulation because of the assumption that the time-sharing sub-intervals are non-overlapping across all users.

Regarding cross-layer design for CRNs, our paper is related with [11,12]. In [12], Tang et al. studied joint spectrum allocation and scheduling problems in cognitive
radio wireless networks with the objective of achieving fair spectrum sharing. In [11], a joint problem of spectrum sharing, scheduling, and routing assignment is investigated for CRN. Compared with these two papers, our main contribution is on the computation of power control and the distributed implementation of the proposed approach.

There have been extensive research efforts on distributed optimization for wireless networks. Some of these algorithms focus on channel assignment problem [13] or power control problem [14], without addressing cross-layer optimization. Research efforts addressing cross-layer optimization problems in distributed fashion include [5,9,15]. In [5], scheduling, power control, and channel assignment are considered within one problem, and a distributed optimization algorithms is proposed. The authors assumed that time-sharing sub-intervals are non-overlapping, thus no interference exists. In [9], a cross-layer optimization problem for CRNs, with a joint consideration of power control, scheduling, and routing, is studied. In [15], a distributive non-cooperative game is proposed to perform channel assignment, adaptive modulation, and power control for multi-cell multi-user OFDMA networks. Compared with these two works, our main contribution is the protection of PUs.

3. NETWORK MODEL AND PROBLEM FORMULATION

We consider a WRAN scenario depicted in Figure 1. The spectrum of interest is divided into $K$ orthogonal channels through multiple access techniques, such as FDMA. There is one WRAN BS, which can access all $K$ channels, at the center of each cell. We consider the downlink scenario in which data are transmitted from WRAN BSs to SUs. We assume that a BS needs exactly one channel to support an SU. The primary network consists of several primary BSs and many PUs who are licensed to use the $K$ orthogonal channels. Each primary BS transmits signals to nearby PUs on an arbitrary channel. Each SU receives signals on a single channel from the nearest primary BS. In the same area, a CRN, which consists of SUs and a group of WRAN BSs, is deployed. Note that in the CRN, SUs cannot introduce noise signals that violate interference constraints for primary network.

We first need to provide some notation. Let the number of SUs be $N$, the number of WRAN BSs be $B$, the number of PUs be $J$, and the number of primary BSs be $C$. Denote the set of SUs as $S = \{s_1, s_2, \ldots, s_N\}$, the set of WRAN BSs as $B = \{b_1, b_2, \ldots, b_B\}$, the set of PUs as $P_u = \{p_{u_1}, p_{u_2}, \ldots, p_{u_J}\}$, and the set of primary BSs as $P_b = \{p_{b_1}, p_{b_2}, \ldots, p_{b_C}\}$. Denote the maximum transmit power as $P_{\text{max}}$. We then introduce an integer parameter $Q$ that represents the total number of power levels to which a transmitter can be adjusted, that is, $1P_{\text{max}}/Q, 2P_{\text{max}}/Q, \ldots, P_{\text{max}}$.  

3.1. Network model

3.1.1. Path loss model.

For a transmission from $b_i$ to PU $p_{u_j}$ and SU $s_j$, a widely used model for propagation gain are shown in Equations (1) and (2), respectively.

$$g_{ij}^* = d_{ij}^{-n}$$  \hspace{1cm} (1)  

$$g_{ij} = d_{ij}^{-n}$$  \hspace{1cm} (2)  

$g_{ij}^*$ and $g_{ij}$ denote the propagation gain from $b_i$ to $p_{u_j}$ and $s_j$, respectively, $d_{ij}^*$ is the physical distance between $b_i$ and $p_{u_j}$, $d_{ij}$ is the physical distance between $b_i$ and $s_j$, and $n$ is the path loss index.

3.1.2. Channel assignment.

A WRAN BS needs exactly one channel to support an SU. For CRN, each SU can be assigned at most one channel associated with a BS, as shown in Equation (3).

$$\sum_{i=1}^{B} \sum_{k=1}^{K} \sum_{q=1}^{Q} x_{ij}^{kq} \leq 1 \ \ j \in \{1, 2, \ldots, N\}$$  \hspace{1cm} (3)  

$x_{ij}^{kq}$ is a binary assignment variable indicating that $b_i$ transmits data to $s_j$ on channel $k$ at the $q$th transmit power level when $x_{ij}^{kq} = 1$ and 0 otherwise.

Regarding scheduling in the frequency domain, one WRAN BS needs exactly one channel to support an SU, as shown in Equation (4).

$$\sum_{j=1}^{N} \sum_{q=1}^{Q} x_{ij}^{kq} \leq 1 \ \ i \in \{1, 2, \ldots, B\} \ \ k \in \{1, 2, \ldots, K\}$$  \hspace{1cm} (4)  

3.1.3. Transmission throughput.

Receiving constraint is considered regarding a successful transmission. Suppose there is a transmission from $b_i$ to $s_j$ on channel $k$ using transmit power $qP_{\text{max}}/Q$. The received power $P_{ij}^{kq}$ can be calculated as in Equation (5).

$$P_{ij}^{kq} = \frac{qg_{ij}P_{\text{max}}}{Q}$$ \hspace{1cm} (5)  

$$i \in \{1, 2, \ldots, B\} \ \ j \in \{1, 2, \ldots, N\}$$  \hspace{1cm} (6)  

$$k \in \{1, 2, \ldots, K\} \ \ q \in \{1, 2, \ldots, Q\}$$  \hspace{1cm} (7)  

The received power at $s_j$ should be no less than a preset transmit threshold power, denoted as $\bar{h}$. Then we can define a channel capacity matrix $A$ as in Equation (10).
Figure 1. An example of wireless regional area network (WRAN)-based cognitive radio networks. BS, base station; PU, primary user; SU, secondary user.

\[
A_{ij}^{kq} = \begin{cases} 
W \times \log_2 \left(1 + \frac{p_{ij}^{kq}}{N_0}ight) & \text{if } qP_{\max}g_{ij} \geq t_i \\
0 & \text{otherwise}
\end{cases} \quad (8)
\]

\[
i \in \{1, 2, \ldots, B\} \quad j \in \{1, 2, \ldots, N\} \quad (9)
\]

\[
k \in \{1, 2, \ldots, K\} \quad q \in \{1, 2, \ldots, Q\} \quad (10)
\]

\(N_0\) denotes the ambient noise power and \(W\) represents the bandwidth of one channel.

### 3.1.4. Interference constraint.

For a link to be interference free from another transmitter, it is required that the received interference power from any transmitter working on that channel should be no greater than a preset threshold value, denoted as \(t_i\). Thus, for \(s_j\) to be able to receive signals on channel \(k\), we derive the following constraint (Equation (11)):

\[
\sum_{P_{ij}^{kq} \geq t_i} x_{ij}^{kq} + \left( I_{ij}^k + \sum_{a=1}^{C} y_{a*j}^k \right) \times \sum_{i=1}^{B} \sum_{q=1}^{Q} x_{ij}^{kq} \leq I_{ij}^k \quad (11)
\]

\[
i \in \{1, 2, \ldots, B\} \quad j \in \{1, 2, \ldots, N\} \quad (12)
\]

\[
k \in \{1, 2, \ldots, K\} \quad q \in \{1, 2, \ldots, Q\} \quad (13)
\]

\(I_{ij}^k\) denotes the cardinality of the set \(\{x_{il}^{kq} | P_{ij}^{kq} \geq t_i, l \neq j, 1 \leq l \leq N, 1 \leq i \leq B, 1 \leq q \leq Q\}\). We introduce another binary constant \(y_{a*j}^k\) indicating the primary BS \(p_{b_j}\) is transmitting on channel \(k\) and interfering with \(s_j\) (the received power from \(p_{b_j}\) at \(s_j\) is no less than \(t_i\) on channel \(k\)) when \(y_{a*j}^k = 1\) and 0 otherwise.

### 3.1.5. Protecting primary users.

To protect PUs from interference, it is required that the received power from secondary links on the same channel that the PU is operating should all be less than the \(t_i\), as shown in Equation (14).

\[
\sum_{c_{j*} \neq k} \sum_{i=1}^{N} x_{il}^{kq} = 0 \quad (14)
\]

\[
i \in \{1, 2, \ldots, B\} \quad j \in \{1, 2, \ldots, N\} \quad e^* \in \{1, 2, \ldots, J\} \quad (15)
\]

\[
k \in \{1, 2, \ldots, K\} \quad q \in \{1, 2, \ldots, Q\} \quad j^* \in \{1, 2, \ldots, J\} \quad (16)
\]

\(P_{ij}^{kq}\) denotes the received power from \(h_i\) to \(p_{u_j}\) on channel \(k\) when the transmit power is \(qP_{\max}/Q\). In addition, \(c_{j*}\) is a binary variable indicating that \(p_{u_j}\) is operating on channel \(k\) when \(c_{j*} = 1\) and 0 otherwise.
3.1.6. Objective.

The objective of our problem is to maximize the total throughput of CRNs, which can be stated as in Equation (17).

\[
\max \sum_{i=1}^{B} \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{q=1}^{Q} x_{ij}^{kq} A_{ij}^{kq}
\]  

(17)

\[i \in \{1, 2, \ldots, B\} \quad j \in \{1, 2, \ldots, N\} \quad k \in \{1, 2, \ldots, K\} \quad q \in \{1, 2, \ldots, Q\}\n
3.2. Problem formulation

Putting together all the constraints described in Section 3.1, we have the following formulation (Section 3.2). The optimization problem is in the form of binary integer programming, which is a non-deterministic polynomial-time hard in general.

\[
\max \sum_{i=1}^{B} \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{q=1}^{Q} x_{ij}^{kq} A_{ij}^{kq}
\]  

(20)

\[s.t. \sum_{i=1}^{B} \sum_{k=1}^{K} \sum_{q=1}^{Q} x_{ij}^{kq} \leq 1\]  

(21)

\[\sum_{j=1}^{N} \sum_{k=1}^{K} x_{ij}^{kq} \leq 1\]  

(22)

\[\sum_{P_{ij}^{kq} \geq l_i} x_{ij}^{kq} + \left( j^k + \sum_{d=1}^{C} y_{d,*}^{k} \right) x_{ij}^{kq} \leq j^k\]  

(23)

\[\sum_{c,j,k} \sum_{l=1}^{N} x_{ij}^{kq} = 0\]  

(24)

\[x_{ij}^{kq} \in \{0, 1\} \quad i \in \{1, 2, \ldots, M\} \quad j \in \{1, 2, \ldots, N\}\]  

(25)

\[k \in \{1, 2, \ldots, K\} \quad q \in \{1, 2, \ldots, Q\} \quad l \in \{1, 2, \ldots, N\}\]  

(26)

\[a \in \{1, 2, \ldots, C\} \quad e \in \{1, 2, \ldots, J\}\]  

(27)

3.3. Fairness considerations

In this paper, we focus on throughput maximization for the CRN. Because the CRN is bandwidth limited and space constrained, it comes down that some SUs may suffer from zero rates. Therefore, when the number of SUs is large, we need to frequently reassign the channels to satisfy fairness requirement. In particular, we can divide the set of SUs into multiple subsets and consider these subsets in a round-robin manner. Given a particular subset of SUs, we can then apply the distributed algorithm to maximize the network throughput.

4. DESIGN OF A DISTRIBUTED OPTIMIZATION ALGORITHM

In this section, we present a distributed optimization algorithm. This algorithm increases the CRN throughput iteratively until it cannot be increased further. The main idea, which includes maximum power calculation, potential throughput gain estimation, BS sorting, and channel usage implementation, is presented in Section 4.1. The details of each module is described in Section 4.2, followed by a complexity analysis in Section 4.3 and convergence proof in Section 4.4.

4.1. Overview

The proposed distributed algorithm increases the overall CRN throughput iteratively and terminates until the overall network throughput cannot be further increased. The proposed distributed algorithm consists of the following steps.

- In the CRN, each WRAN BS maintains the sum of the capacity of all links connected with itself. We assume that each WRAN BS also maintains a table of local information (TLI) of PUs and SUs. In particular, each WRAN BS first obtains the knowledge of positions of the local SUs and PUs. We assume each SU acquires its position through global positioning system and sends it to the WRAN BS. It is also assumed that PUs subscribe to primary network service providers and register their positions at primary BSs. Therefore, each WRAN BS can obtain the positions of PUs by communicating with primary BSs. For those PUs and SUs that have already been assigned channels, the WRAN BS also understands their associated transmit power settings as well as their channel usage pattern.

- After TLI establishment, each WRAN BS estimates the maximum throughput it can produce, which includes power control calculation and potential throughput gain estimation. Before estimating potential maximum throughput gain, each BS pretends that all existing assigned links connected to itself are annihilated. Each WRAN BS calculates its possible maximum transmit power on each channel to avoid interfering with other existing links. Then each WRAN BS proceeds in calculating the maximum throughput produced by itself under the interference constraints imposed by both the primary network and CRN.
After estimating the maximum throughput, each WRAN BS exchanges its result with neighboring WRAN BSs, and finally, the largest is identified as associated with one WRAN BS. When there are multiple WRAN BSs with the same largest cell throughput, we break the tie deterministically based on their cell IDs. The benefit of the deterministic approach is that a unique WRAN BS is always picked from the proposed distributed algorithm. Then the chosen WRAN BS is required to implement its channel assignment and transmit power setting to best increase the overall network throughput.

During the last step of each iteration, the changes of channel assignment and power control information should be updated in the TLIs of neighboring WRAN BSs.

The basic diagram of the distributed algorithm is shown in Figure 2.

4.2. Details of each module

Before we present the details of the distributed algorithm, we first introduce the following notation. Let $l^k_{ij}$ denote the link between $b_i$ and the $s_j$ on channel $k$. We assume $UB^k_q$ denotes the maximum allowed transmit power that $b_q$ can utilize on channel $k$ without interfering all existing links in neighboring cells that operate on the same channel. Recall that $P_{\text{max}}$ is the maximum transmit power. During an iteration, we may find that, under the interference constraint, $UB^k_q$ is usually smaller than $P_{\text{max}}$. We use $UB^k_q$ for this purpose, where the acronym $UB$ indicates the current upper bound on the transmit power.

We define the excluded channel set as the set of channels that cannot be assigned to one SU, that is, $s_{qj}$, denoted as $\Omega_j$. $n_r$ denotes the current set of unconnected SUs. $q_i$ denotes the $i$th SU within the cell of $b_q$, $r_{jk}$ denotes the throughput gain of the channel $k$ when assigned to the $j$th element of the current set of unconnected SUs. $y_{jk}$ is a binary assignment variable indicating the $k$th channel is assigned to the $j$th element of the current set of unconnected SUs when $y_{jk} = 1$ and 0 otherwise.

4.2.1. Table of local information establishment.

We first present the method of TLI establishment. For each WRAN BS, the TLI only records the positions of PUs and SUs, channel usage pattern, and associated transmit power settings within the cell as well as within neighboring cells. When one WRAN BS tends to assign one channel, it also entails information of already existing local links, in terms of positions of user nodes and associated transmit powers, to carefully avoid causing interference.

4.2.2. Maximum power calculation.

We now present the method in the module of maximum power calculation. The maximum transmit power for any assigned link is always bounded by $P_{\text{max}}$. The effort to achieve the maximum transmit power agrees perfectly well with the objective to produce the maximum cell throughput. In particular, for each cell, the WRAN BS operates at certain levels of transmit powers on each channel. In order to produce the largest cell throughput, the WRAN BS would boost its transmit power on all channels to the point that existing links are about to be interfered.

The procedure of maximum power calculation algorithm is to browse all links in both CRN and primary network in neighboring cells that operates on the same channel and

![Figure 2](image-url)
chooses the smallest upper link power. The implementation of the module should follow the algorithm presented in Table I. $\lfloor z \rfloor$ represents the maximum power level that is less than the value of $z$.

4.2.3. Excluded channel sets recording.

Subsequently, we present the method of recording excluded channel set. This module is necessary because it is possible that some SUs in the cell cannot be supported because of an already too much interference noise on certain channels. In other words, even if the transmit power is set as $P_{\text{max}}$, the link still cannot conduct successful transmission because the signal-to-interference-plus-noise ratio requirement is not satisfied.

As mentioned in Section 4.1, after calculating the upper bound power for each channel, each WRAN BS should also be aware that some channels cannot be assigned to certain SUs within the cell because of too much interference noise. Thus, we assume each WRAN BS records excluded channel sets associated with SUs following the algorithm presented in Table II. $P_{\text{r}_i j}$ denotes the received power from $p_{b_j}$ to $s_j$.  

4.2.4. Maximum cell throughput estimation.

We then present the method for each WRAN BS to estimate the maximum cell throughput. Being aware that some channels cannot be assigned to certain SUs, the WRAN BS sets out to assign each channel to the SUs within the cell after calculating the maximum transmit powers. This problem can be formulated as the assignment problem, which is one of the fundamental combinatorial optimization problems in the branch of optimization or operations research in mathematics. It consists of finding a maximum weight matching in a weighted bipartite graph. The assignment problem can be solved by the famous Hungarian algorithm.

<table>
<thead>
<tr>
<th>Table II. Excluded channel sets recording.</th>
</tr>
</thead>
</table>
| r = \[
\begin{bmatrix}
    r_{11} & r_{12} & \cdots & r_{1K} \\
    r_{21} & r_{22} & \cdots & r_{2K} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{n_1} & r_{n_2} & \cdots & r_{n_K}
\end{bmatrix}
\] (28) |

Now the problem of maximum cell throughput estimation can be formulated as in Equation (29).

$$\max \sum_{j=1}^{n_r} \sum_{k=1}^{K} r_{jk}y_{jk}$$

s.t. $\sum_{j=1}^{n_r} y_{jk} \leq 1$  
$k \in \{1, 2, \ldots, K\}$

$$\sum_{k=1}^{K} y_{jk} \leq 1$$  
$j \in \{1, 2, \ldots, n_r\}$

This problem can be formulated as the assignment problem, which can be solved by the famous Hungarian algorithm. We now present the Hungarian algorithm [16] to assign channels within each cell, independent to what happens in the rest, as follows:

- **Step 1:** If $r$ is not a square matrix (there are more channels than local SUs or conversely), we have to augment $r$ into a square matrix by adding zero rows or columns.
- **Step 2:** Multiply the matrix $r$ by $-1$.
- **Step 3:** Subtract the minimum value of each row from row entries.

<table>
<thead>
<tr>
<th>Table I. Maximum power calculation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 for each channel $k$,</td>
</tr>
<tr>
<td>2 $UB_k = \infty$,</td>
</tr>
<tr>
<td>3 for each existing link $l_{ij}^k$,</td>
</tr>
<tr>
<td>4 if $UB_k &gt; \frac{P_{\text{r}_{ij}}}{\gamma_d}$,</td>
</tr>
<tr>
<td>5 $UB_k = \frac{P_{\text{r}_{ij}}}{\gamma_d}$,</td>
</tr>
<tr>
<td>6 end if</td>
</tr>
<tr>
<td>7 end for</td>
</tr>
<tr>
<td>8 for each PU $p_{ij}$,</td>
</tr>
<tr>
<td>9 if $UB_k &gt; \frac{P_{\text{r}<em>{ij}}}{\gamma_d}$ and $c</em>{ij} = 1$,</td>
</tr>
<tr>
<td>10 $UB_k = \frac{P_{\text{r}_{ij}}}{\gamma_d}$,</td>
</tr>
<tr>
<td>11 end if</td>
</tr>
<tr>
<td>12 end for</td>
</tr>
<tr>
<td>13 if $UB_k &gt; P_{\text{max}}$,</td>
</tr>
<tr>
<td>14 $UB_k = P_{\text{max}}$,</td>
</tr>
<tr>
<td>15 end if</td>
</tr>
<tr>
<td>16 end for</td>
</tr>
<tr>
<td>17 return UB</td>
</tr>
</tbody>
</table>

SU, secondary user; BS, base station.
• Step 4: Subtract the minimum value of each column from column entries.
• Step 5: Select rows and columns across in which you will draw lines, such that all zeros are covered and that no more lines will have been drawn than necessary.
• Step 6: If the number of the lines equals the number of rows, choose a combination of zero elements from the modified gain matrix such that the position of each chosen element is incidentally on a unique row and column. Then the optimal assignment result will consist of the channel–SU pairs as represented by the chosen elements in the modified gain matrix. If the number of the lines is less than the number of rows, go to step 7.
• Step 7: Find the smallest element that is not covered by any of the lines. Then subtract it from each entry that is not covered by the lines, and add it to each entry which is at the intersection of a vertical and horizontal line. Go back to step 5.

4.2.5. Base station sorting.

This module aims to find the maximum cell throughput and hence the network throughput can be increased greedily. The implementation of this module entails a certain amount of information exchange. After each WRAN BS is associated with a maximum cell throughput, they can exchange their results with neighboring WRAN BSs in a distributed fashion. Once a WRAN BS receives its knowingly best cell throughput, it propagates this datum to its neighboring WRAN BSs exactly once. In particular, each WRAN BS is only concerned if its maximum cell throughput is larger than any other WRAN BS in this iteration. Thus, they would discard their own maximum cell throughput along with the associated channel usage information once they realize some other WRAN BSs produce larger cell throughput. For the case of equal cell throughput, the WRAN BS would also discard its own results if the WRAN BS that produces the same amount of cell throughput is indexed smaller. This sorting procedure terminates when any WRAN BS has not been notified of any larger cell throughput for a preset amount of time.

4.2.6. Channel usage implementation.

Finally, we discuss the method of channel usage implementation. This module is only applied at the WRAN BS whose maximum cell throughput is the largest among all WRAN BSs in each iteration. The WRAN BS implements the calculated channel assignment and transmit power settings. This WRAN BS also has to inform its neighboring BSs to update their TLIs for calculating the maximum transmit powers in the next iteration.

4.3. Complexity analysis

As for the distributed scheme, the modules that do the majority computation include estimating the maximum transmit power, recording the excluded channel sets, and calculating the cell throughput. For the module of maximum transmit power estimation, we mainly investigate the number of loops in Table I. We suggest that the main complexity factor is the number of SUs $N$, whereas $B$, $C$, and $J$ are all of $O(1)$. Because the procedure between lines 2 and 15 can be iterated at most $(BN + J)$, the computation for the module can be as much as $O((BN + J) \times C) = O(N)$. Consider the module of recording the excluded channel sets, the procedure is iterated $O(n_r(N - n_r))$ according to Table II. Thus, the computational complexity can be $O(N^2)$. Then, we look at the module of calculating the cell throughput. Based on [16], the assignment problem proposed by the module can be solved within running time $O(\max\{C, n_r\}^4)$. It can be inferred that the computation for one BS during one iteration can be calculated as $O(N^4)$. As only one BS is required to implement its channel assignment and power control in one iteration, the total number of iterations is measured as $O(B)$. Therefore, the overall computational complexity for the distributed scheme at one BS is $O(B \times (N + N^2 + N^4)) = O(BN^4) = O(N^4)$, which is much less than the optimal algorithm.

4.4. Convergence behavior

We now show that the algorithm must converge. We show that in each iteration, the algorithm increases the throughput performance of CRN. Because the overall throughput is upper bounded, this implies that the algorithm must converge.

We first denote the sum rates of the links connected to $b_i$ as $T_i$. Then we obtain Equation (30).

$$T_i = \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{q=1}^{Q} \bar{X}_{ij}^{kq} \bar{A}_{ij}^{kq} \quad i \in \{1, 2, \ldots, B\}$$

Thus, the objective of our throughput maximization problem is $\sum_{i=1}^{B} T_i$. At the beginning of each iteration, each WRAN BS’s configuration represents a feasible solution to the maximum throughput problem as shown in Section (3.2). After solving the maximum throughput estimation problem as shown in Equation (29), $b_i$ obtains the optimal $T_i$. As a result, the new value of $T_i$ must be no less than the previous iteration. Subsequently, one WRAN BS $b_i$ is chosen to implement its result while keeping links to other WRAN BSs protected, which means that the value of $T_j$ ($j \neq i$) remain the same. Therefore, no matter which BS is chosen to implement new settings for channel assignment and power control, the network throughput is expected to grow larger than the lower bound at the beginning of the iteration. Because the network throughput performance monotonically increases after every iteration, convergence of the greedy algorithm is guaranteed.
5. OTHER RELATED ALGORITHMS

5.1. Optimal algorithm

Because the problem formulation (Section 3.2) falls into the binary integer programs, it can be solved by the branch-and-bound algorithm [3] that yields the optimal solution.

The complexity analysis of the optimal algorithm is presented as follows. For the channel assignment and power control problem, the solution space contains all combinations of $2^{BNKQ}$ binary variables. Thus the optimal algorithm could potentially search all $2^{BNKQ}$ binary integer vectors, and the running time is $O(2^{BNKQ})$.

5.2. Two-phased algorithm

In [8], a problem of maximizing network throughput for CRN is studied. A two-phased scheme is proposed to control the transmit power of BSs and assign channels. In the first phase, a distributed power updating process is employed to maximize the coverage of the network. In particular, the maximum transmit powers of BSs on all channels are sought to avoid interfering with PUs. In the second phase, centralized channel assignment is carried out within the cognitive network to maximize its throughput. To specify, given the maximum transmit powers of BS on all channels, the channel assignment problem can be transformed into finding a maximum weighted matching from a weighted bipartite graph. The two-phased scheme has a complexity of $O(N^4)$.

5.3. Dynamic interference graph allocation

In [7], Hoang and Liang proposed the dynamic interference graph allocation (DIGA) that implements power control and channel assignment to maximize coverage for CRNs. In the DIGA scheme, a channel is allocated to one SU at a time, until either all SUs are served, or there is no more feasible assignment. At each iteration, channel assignment and power control should be carefully implemented so that any prior established links and all PUs are protected. According to DIGA, to establish a link between one WRAN BS and an SU on channel $k$ with a certain power level, a penalty value is defined as the total number of unserved SUs that cannot be assigned the same channel anymore. Thus, the smallest penalty value is sought in each iteration to iteratively implement channel assignment and power control. As for our work, because the objective is to maximize the capacity of the CRN, we redefine the definition of penalty value to be the total capacity loss associated with unserved SUs on the specific channel. After this minor adaptation, we compare the performance of the proposed distributed algorithm with DIGA in Section 6. The complexity of DIGA is $O(CN^5 + CN^2 J) = O(CN^5) = O(N^5)$ [7].

5.4. Power-based algorithm

In [17], the problem of allocating channels is studied to satisfy the rate requirements of the application while the total transmit power is minimized. The proposed power-based scheme is also an iterative approach, similar as DIGA [8]. The difference lies in the definition of the penalty value which is defined as the increase in the total transmit power of links associated with channel $k$ if this channel is assigned with a certain transmit power. This scheme is termed as Minimum Incremental Power Allocation (MIPA). The complexity of the MIPA scheme is $O(CN^5 + CN^2 J) = O(CN^5) = O(N^5)$, which is at the same order as the complexity of DIGA scheme [17].

6. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed distributed algorithm through simulations. We compute optimal solutions using CPLEX 9.0 (ILOG SA, Gentilly, France) [18]. We compare the results of distributed algorithm with the globally optimal method.

6.1. Simulation setup

We consider a square service area of size 100 × 100 km in which a CRN is deployed. Four WRAN BSs are deployed at the centers of four square sub-areas, as shown in Figure 1. We consider one random scenario of primary network, with coordinates of primary BSs shown in Table III and PUs shown in Table IV. SUs are randomly deployed across the entire service area with uniform distribution. A sample network is shown in Figure 1. The ambient noise power at each PU and SU is $N_0 = 5 \times 10^{-11}$ W. The number of channels is $K = 4$. We establish the primary network as follows. Each of the four primary BSs choose different channels to serve the PUs. The transmit power for all primary BSs is set as 20 mW. According to the physical distance to each primary BS, each PU is covered by the nearest primary BS.

Table III. Node coordinates of four primary base stations.

<table>
<thead>
<tr>
<th>$p_b_i$</th>
<th>$(x_i, y_i)$ (in meters)</th>
<th>$p_u_i$</th>
<th>$(x_i, y_i)$ (in meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(95970, 75130)</td>
<td>3</td>
<td>(34040, 25510)</td>
</tr>
<tr>
<td>2</td>
<td>(58530, 50600)</td>
<td>4</td>
<td>(22380, 69910)</td>
</tr>
</tbody>
</table>

Table IV. Node coordinates of 10 primary users.

<table>
<thead>
<tr>
<th>$p_u_i$</th>
<th>$(x_i, y_i)$ (in meters)</th>
<th>$p_u_i$</th>
<th>$(x_i, y_i)$ (in meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(65570, 50600)</td>
<td>6</td>
<td>(85770, 82350)</td>
</tr>
<tr>
<td>2</td>
<td>(13570, 63180)</td>
<td>7</td>
<td>(94310, 69480)</td>
</tr>
<tr>
<td>3</td>
<td>(84910, 67690)</td>
<td>8</td>
<td>(39220, 31710)</td>
</tr>
<tr>
<td>4</td>
<td>(33400, 74620)</td>
<td>9</td>
<td>(65560, 55020)</td>
</tr>
<tr>
<td>5</td>
<td>(57870, 49710)</td>
<td>10</td>
<td>(27120, 23440)</td>
</tr>
</tbody>
</table>
6.2. Simulation results

In this section, we provide simulation results by comparing the distributed algorithm with other algorithms. In particular, we look into the impacts of four different system parameters, the number of SUs $N$, the transmit threshold power $t_t$, the interference threshold power $t_i$, and the number of power levels $Q$. We vary each of the four system parameters while keeping the others unchanged to produce different parameter settings. For each set of system parameters, we generate 100 instances of different deployments of SUs to obtain the average performance. Totally, five schemes are considered, that is, the global optimal scheme, the proposed distributed scheme, the two-phase algorithm, the DIGA scheme, and the MIPA scheme. The simulation results are discussed next.

6.2.1. The impact of transmit threshold power.

In Figure 3, we look at the impact of transmit threshold power $t_t$ on the throughput of CRN. We compare the performance of the optimal algorithm, the distributed algorithm, the two-phase algorithm, the DIGA scheme, and the MIPA scheme. As can be observed from Figure 3, the global optimal scheme gives the best performance, whereas the proposed distributed scheme consistently outperforms the two-phase algorithm, the DIGA scheme, and the MIPA scheme. The performance gain for the proposed distributed algorithm over the other schemes is mainly due to three reasons. First, the proposed distributed scheme directly addresses the objective of maximizing the throughput in each iteration. The two-phase algorithm tries to maximize transmit powers of BSs on all channels in the first phase, which might not best serve the interest of maximizing the network throughput. The DIGA and MIPA schemes introduce the penalty value of potential throughput loss, transmit power increase for each channel assignment with a power level, and seek to minimize the value in each iteration. They ignore the fact that the newly added link probably will not bring the maximal incremental throughput, which does not agree with the objective from a greedy perspective. Second, the proposed distributed algorithm uses transmit power more efficiently. Under the distributed algorithm, each BS boosts its transmit power to provide higher rates for SUs. Whereas for the two-phase algorithm, after WRAN BSs set their high transmit power the first phase, SUs may not be able to access downlink channels because of high interference. Third, the proposed distributed algorithm is implemented by each BS, bringing the maximal incremental throughput by establishing links with multiple SUs at the same time. The DIGA and MIPA schemes only establish one link at each step, which probably generates less benefits in terms of maximizing overall throughput.

It can be observed that the overall throughput monotonically decreases as $t_t$ is enlarged. The reason behind is very simple: higher threshold power decreases the number of feasible links.

6.2.2. The impact of interference threshold power.

Figure 4 investigates the impact of interference threshold power on the throughput of CRN. The main trend is that the overall throughput increases as the interference threshold power is enlarged. This can be explained by the fact that larger interference threshold power generates more opportunities for more links to be active simultaneously, thus bringing throughput increase to the overall performance. It can also be noted that the proposed distributed algorithm outperforms the two-phase algorithm, the DIGA scheme, and the MIPA scheme consistently.
6.2.3. The impact of the number of secondary users.

Figure 5 shows the impact of the number of SUs on the throughput of CRN. The main trend is that the total throughput is increased as the number of SUs increases. The rationale behind is that more SUs tend to render more opportunities to establish links with higher capacity, that is, the WRAN BSs are closer to their associated SUs. It should also be noted that the proposed distributed scheme yields better performance than the other three schemes.

6.2.4. The impact of the number of power levels.

Figure 6 depicts the impact of the number of power levels on the throughput of CRN. It is obvious that the number of power levels will increase the throughput performance of CRN. With larger number of power levels, the transmit power can be more finely controlled; thus, it is probable that multiple links on the same channel can be active simultaneously. However, when \( Q \) is large enough, the plots do not obviously increase because at this point larger \( Q \) will have little impact on the assignment result, that is, increase the number of assigned channels. Furthermore, it can be also observed that the proposed distributed scheme produce better solutions in terms of total throughput of CRN than other three schemes.

7. DISCUSSION

The reason that the distributed algorithm demonstrates good performance is due to the optimization of a large set of variables. A detailed investigation at Equation (29) and Section (3.2) would reveal the truth. During each iteration, the distributed algorithm performed optimization on a set of variables that shared the same attributes, that is, associated with the same WRAN BS. After each iteration, many variables could be determined. Let’s consider the extreme case. If only one WRAN BS was available, the distributed algorithm would yield the optimal solution. In contrast, the two-phase scheme overboosted the transmit powers of WRAN BSs during the first phase, which harmed the overall performance by causing too much interference. The DIGA and MIPA scheme only determined one variable using greedy strategy during each iteration, which was probably not in the interest of the overall performance of the CRN. Besides, although some variables associated with one user node can be set to 0 at each step, many more variables were still unspecified than the distributed algorithm. From the foregoing, the advantage of the distributed algorithm should be attributed to the manipulation of large set of variables at each step.

The distributed algorithm performs satisfyingly close to the optimal solution as shown in Section 6. The algorithm takes advantage of the computing ability of WRAN BSs and assigning a considerably smaller computing task to each WRAN BS. Fortunately, each computing task also involves a very large set of variables. All the facts mentioned support the advantage of the distributed algorithm.

As future work, the network model and problem formulation can be changed in many different dimensions. For instance, the primary network can be cooperative with the CRN. As another example, the behavior on other network layers, such as scheduling on the media access control layer, can be considered. In addition, other variables can be evaluated in terms of the impact on the overall performance, such as the number of time slots. Moreover, the objective of the problem can be varied because of different application scenarios. For example, it is more desirable under certain circumstances to investigate the fairness among SUs instead of the overall performance of CRN.

8. CONCLUSION

In this paper, we investigate the cross-layer design and distributed optimization algorithm for a CRN. We first
develop a mathematical model for such problem with a joint consideration of power control and channel assignment. The main contribution of this paper is the development of a distributed optimization algorithm that iteratively increases the overall CRN network throughput. This algorithm consists of several modules, including maximum power calculation, cell throughput estimation, BS sorting, and channel usage implementation. Through simulation results, we compare the performance of the distributed optimization algorithm with the other algorithms and validate its efficacy. In addition, we also give some insights on the advantage of the distributed algorithm over other algorithms and describe future work.

ACKNOWLEDGEMENTS

This work was supported in part by the National Science Foundation (NSF) through award ECS-0725522. We would also like to thank the anonymous reviewers for their comments that helped us improve the paper.

REFERENCES


AUTHORS’ BIOGRAPHIES

Wenxuan Guo received his B.S. degree in Computer Science and Technology and M.S. degree in Information Security from Huazhong University of Science and Technology, Wuhan, China in 2004 and 2007, respectively. He is currently a Ph.D. student in Electrical and Computer...
Engineering at Worcester Polytechnic Institute, Worcester, MA, USA. His research interest is on optimization of wireless networks.

Xinming Huang received his Ph.D. in Electrical Engineering from Virginia Tech, Blacksburg, VA, USA in 2001. He was formerly a member of the Technical Staff in Bell Labs of Lucent Technologies from 2001 to 2003. He is currently an associate professor in the Department of Electrical and Computer Engineering at the Worcester Polytechnic Institute. His researches are in the areas of circuit design and system architecture.